Confidence based Consensus Model for Intuitionistic Fuzzy Preference relations

Raquel Ureña*, Francisco Chiclana†, Hamido Fujita‡ Enrique Herrera-Viedma§
†Department of Informatics Engineering, University of Cadiz, Cadiz Spain E-mail: raquel@decsai.ugr.es
†Centre for Computational Intelligence (CCI), Faculty of Technology De Montfort University, Leicester, UK
E-mail: chiclana@dmu.ac.uk
‡Iwate Prefectural University, Takizawa, Iwate, Japan E-mail: issam@iwate-pu.ac.jp
§Department of Computer Science and Artificial Intelligence University of Granada, Granada, Spain Email: viedma@decsai.ugr.es

***Abstract*—Intuitionistic fuzzy preference relation are gaining increasing relevance in the field of group decision making as they provide the experts with means to allocate the uncertainty inherent in their proposed opinions. A key issue in this field is to reach a solution accepted by the majority of the members of the group. In this contribution we present a new confidence-consistency based consensus model. Moreover to rank the alternatives we present the implementation of Orlovsky's non-dominance concept to define the fuzzy quantifier guided non-dominance choice degree for intuitionistic fuzzy preference relations.**

I. Introduction

In Group Decision Making situations, GDM, Intuitionistic fuzzy preference relations, IFPRs, based on Attanasov's Intuitionistc Fuzzy sets, [1], suppose an interesting framework for the experts to express their judgements, since they allow them to allocate certain levels of uncertainty in their opinions.

On the other hand, a key issue in GDM consists on achieving a full and unanimous agreement among all the experts. However in the majority of the occasions is not reachable in practice. An alternative approach is to use softer consensus measures [3] that better represent the human perception of the essence of consensus. These approaches define the consensus process as a dynamic and iterative group discussion coordinated by a moderator that helps experts to bring their opinions closer. To guide the consensus process different indicators has been used in the literature. Among them we can highlight twofold: Consistency and Similarity. Consistency is linked to rationality of individuals whereas similarity can be interpreted as a measure of general or widespread agreement. By combining both consistency and similarity functions, Herrera-Viedma et al. [10] developed a feedback mechanism to provide advice to experts in order to increase the consensus level of the group. Futhermore, Chiclana et al. in [5] designed a two stage model with a first stage aiming to reach acceptable consistency level while the second one was used to achieve a predefined consensus level. Focusing on the case of IFPRs there are already available some consensus models in the literature [24], [25].

However in environments where the experts present high level of uncertainty in their opinions, other measures should be taken into account as well, to guide the consensus process. In this sense, It has been found that freely interacting groups choose the positions of their most confident members as their group decisions. This phenomenon has been witnessed with groups discussing a mathematical puzzle [14], a recall task [18] and a recognition task [13], concluding that confidence was a significant predictor of influence. Furthermore Guha et al. state in [9] that in any real field decision making situation when experts give their responses to a particular alternative, their confidence level regarding the opinions are very much important. In this sense, in [21] it has been presented an approach which asses the experts degree of confidence directly from the experts opinions expressed by means of IFPRs, and so it allows to take into account this valuable information in the decision making process.

The main objective of this contribution is to present a new confindence-consistency based consensus model that takes into consideration both the consistency, and experts' confidence levels to implement a feedback mechanism to support experts to change some of their preference values using simple advice rules that aim at increasing the level of agreement while, at the same time, keeping a high degree of consistency.

The rest of the paper is set out as follows: Section II presents the main mathematical frameworks for representing preferences and the basics concepts needed throughout the rest of the paper. In Section III we present the new Confindence-Consistency based consensus model and a afterwards a new fuzzy quantifier guided non-dominance choice degree for intuitionistic fuzzy preference relations. Finally, Section IV draws conclusions and presents some future work.

II. Background

In group decision making problems, once the set of feasible alternatives (X) is identified, experts are called to express their opinions or preferences on such set. Different preference elicitation methods were compared in [15], where it was concluded that pairwise comparison methods are more accurate than non-pairwise methods because they allow the expert to focus on two alternatives at a time. A comparison of two alternatives by an expert can lead to the preference of one alternative to the other or to a state of indifference between them. Obviously, there is the possibility of an expert being unable to compare them. Two main mathematical models based on the concept of preference relation can be used in this context. In the first one, a preference relation is defined for each one of the above three possible preference states mentioned above (preference, indifference, incomparability) [7], which is usually referred to as a preference structure on the set of alternatives [16]. The second one integrates the three possible preference states into a single preference relation [2]. In this paper, we focus on the second one as per the following definition:

Definition 1 (Preference Relation)**.** *A preference relation P on a set X is a binary relation $\mu_P : X \times X \to D$, where D is the domain of representation of preference degrees provided by the decision maker.*

A preference relation P may be conveniently represented by a matrix $P = (p_{ij})$ of dimension $\#X$, with $p_{ij} = \mu_P(x_i, x_j)$ being interpreted as the degree or intensity of preference of alternative x_i over x_j. The elements of P can be of a numeric or linguistic nature, i.e., could represent numeric or linguistic preferences, respectively. The main types of numeric preference relations used in decision making are: crisp preference relations, additive preference relations, multiplicative preference relations, interval-valued preference relations and intuitionistic preference relations. A comprehensive survey of them have been reported on [27], which the reader is encouraged to consult for further particulars. In this contribution, the focus is on fuzzy preference relations and intuitionistic fuzzy preference relations.

A. Fuzzy Set and Fuzzy Preference Relation

Definition 2 (Fuzzy Set)**.** *Let U be a universal set defined in a specific problem, with a generic element denoted by x. A fuzzy set X in U is a set of ordered pairs:*

$$X = \{(x, \mu_X(x)) | x \in U\}$$

where $\mu_X : U \to [0,1]$ is called the membership function of A and $\mu_X(x)$ represents the degree of membership of the element x in X.

The degree of non-membership of the element x in X is here defined as $\nu_X(x) = 1 - \mu_X(x)$. Thus, $\mu_X(x) + \nu_X(x) = 1$.

Definition 3 (Fuzzy Preference Relation)**.** *A fuzzy preference relation $R = (r_{ij})$ on a finite set of alternatives X is a fuzzy relation in $X \times X$ that is characterised by a membership function $\mu_R : X \times X \longrightarrow [0,1]$ with the following interpretation:*

- *$r_{ij} = 1$ indicates the maximum degree of preference for x_i over x_j*
- *$r_{ij} \in]0.5, 1[$ indicates a definite preference for x_i over x_j*
- *$r_{ij} = 1/2$ indicates indifference between x_i and x_j*

When

$$r_{ij} + r_{ji} = 1 \ \forall i, j \in \{1, \ldots, n\}$$

is imposed the fuzzy preference relation is called reciprocal.

1) Consistency of fuzzy preference relations: Consistency of fuzzy preference relations has been modeled using the notion of transitivity in the pairwise comparison among any three alternatives: if x_i is preferred to x_j ($x_i \succ x_j$) and this one to x_k ($x_j \succ x_k$) then alternative x_i should be preferred to x_k ($x_i \succ x_k$), which is normally referred to as *weak stochastic transitivity* [4]. Any property that guarantees the transitivity of the preferences is called a consistency property. Clearly, the lack of consistency in decision making can lead to inconsistent conclusions; that is why it is crucial to study conditions under which consistency is satisfied [17].

Different properties or conditions have been suggested as rational conditions to be verified by a consistent fuzzy preference relation [4], [12]: triangle condition, weak transitivity, max-min transitivity, max-max transitivity, restricted max-min transitivity, restricted max-max transitivity, additive transitivity, and multiplicative transitivity. In this contribution we focus on Tanino's Multiplicative transitivisty property to model consistency.

Definition 4 (Multiplicative transitivity [20])**.** *A fuzzy preference relation $R = (r_{ij})$ on a finite set of alternatives X is multiplicative transitive if and only if*

$$r_{ij} \cdot r_{jk} \cdot r_{ki} = r_{ik} \cdot r_{kj} \cdot r_{ji} \quad \forall i, k, j \in \{1, 2, \ldots n\} \tag{1}$$

Multiplicative consistency property (1) can be used to estimate the preference value between a pair of alternatives (x_i, x_j) with $(i < j)$ using another different intermediate alternative x_k $(k \neq i, j)$ as follows:

$$mr_{ij}^k = \frac{r_{ik} \cdot r_{kj} \cdot r_{ji}}{r_{jk} \cdot r_{ki}} \tag{2}$$

as long as the denominator is not zero. We call mr_{ij}^k the partially multiplicative transitivity based estimated fuzzy preference value of the pair of alternatives (x_i, x_j) obtained using the intermediate alternative x_k.

The average of all possible partially multiplicative transitivity based estimated values of the pair of alternatives (x_i, x_j) can be interpreted as their global multiplicative transitivity based estimated value

$$mr_{ij} = \frac{\sum_{k \in R_{ij}^{01}} mr_{ij}^k}{\# R_{ij}^{01}};$$

where $R_{ij}^{01} = \{k \neq i, j | (r_{ik}, r_{kj}) \notin R^{01}\}$, $R^{01} = \{(1,0), (0,1)\}$, and $\# R_{ij}^{01}$ is the cardinality of R_{ij}^{01}. Therefore, given a fuzzy preference relation, $R = (r_{ij})$, the following multiplicative transitivity based fuzzy preference relation, $MR = (mr_{ij})$, can be constructed. Notice that when a fuzzy preference relation $R = (r_{ij})$ is multiplicative transitive then $R = MR$. Indeed, if R is multiplicative transitive then (1) holds $\forall i, j, k$. In particular, we have

$$r_{ij} = \frac{r_{ik} \cdot r_{kj} \cdot r_{ji}}{r_{jk} \cdot r_{ki}};$$

whenever $k \in R_{ij}^{01}$. Consequently, $mr_{ij}^k = r_{ij}$ for all i, j and $k \in R_{ij}^{01}$, which proves that $r_{ij} = mr_{ij}$ for all i, j. A fuzzy preference relation R will be referred to as multiplicative consistent from now on when $R = MR$.

Definition 5 (Multiplicative Consistency)**.** *A fuzzy preference relation $R = (r_{ij})$ is multiplicative consistent if and only if $R = MR$.*

The similarity between the values r_{ij} and mr_{ij} is proposed to be used in measuring the level of consistency of a fuzzy preference relation at its three different levels: pair of alternatives, alternatives and relation [11]:

Level 1. *Consistency Index of pair of alternatives.*

$$CL_{ij} = 1 - d(r_{ij}, mr_{ij}) \quad \forall i, j.$$

Here $d(r_{ij}, mr_{ij})$ represents the distance between the values r_{ij} and mr_{ij}. Obviously, the higher the value of CL_{ij} the more consistent is r_{ij} with respect to the rest of the preference values involving alternatives x_i (row i of the fuzzy preference relation) and x_j (column j of the fuzzy preference relation).

Level 2. *Consistency Level of alternatives.*

$$CL_i = \frac{\sum_{j=1;\ i \neq j}^{n} CL_{ij}}{n-1}.$$

Level 3. *Consistency Level of a fuzzy preference relation.*

$$CL = \frac{\sum_{i=1}^{n} CI_i}{n}.$$

The following result characterises multiplicative consistency of a fuzzy preference relation using its corresponding consistency level.

Proposition 1. *A fuzzy preference relation R is multiplicative consistent if and only if $CL = 1$.*

B. Intuitionistic Fuzzy Set and Intuitionistic Fuzzy Preference Relation

The concept of an *Intuitionistic Fuzzy Set* (IFS) was introduced by Atanassov in [1]:

Definition 6 (Intuitionistic Fuzzy Set)**.** *An intuitionistic fuzzy set X over a universe of discourse U is given by*

$$X = \Big\{ \big(x, \langle \mu_X(x), \nu_X(x) \rangle \big) \big| x \in U \Big\}$$

where $\mu_X : U \to [0,1]$, and $\nu_X : U \to [0,1]$ verify

$$0 \leq \mu_X(x) + \nu_X(x) \leq 1 \quad \forall x \in U.$$

$\mu_X(x)$ and $\nu_X(x)$ represent the degree of membership and degree of non-membership of x in X, respectively.

An intuitionistic fuzzy set becomes a fuzzy set when $\mu_X(x) = 1 - \nu_X(x)$ $\forall x \in U$. However, when there exists at least one value $x \in U$ such that $\mu_X(x) < 1 - \nu_X(x)$, an extra parameter has to be taken into account when working with intuitionistic fuzzy sets: the hesitancy degree, $\tau_X(x) = 1 - \mu_X(x) - \nu_X(x)$, that represents the amount of lacking information in determining the membership of x to X. If the hesitation degree is zero, the reciprocal relationship between membership and non-membership makes the latter one unnecessary in the formulation as it can be derived from the former.

Szmidt and Kacprzyk in [19] defined the intuitionistic fuzzy preference relation as a generalisation of the concept of fuzzy preference relation.

Definition 7 (Intuitionistic Fuzzy Preference Relation)**.** *An intuitionistic fuzzy preference relation B on a finite set of alternatives $X = \{x_1, \ldots, x_n\}$ is characterised by a membership function $\mu_B \colon X \times X \to [0,1]$ and a non-membership function $\nu_B \colon X \times X \to [0,1]$ such that*

$$0 \leq \mu_B(x_i, x_j) + \nu_B(x_i, x_j) \leq 1 \quad \forall (x_i, x_j) \in X \times X.$$

with $\mu_B(x_i, x_j) = \mu_{ij}$ interpreted as the certainty degree up to which x_i is preferred to x_j; and $\nu_B(x_i, x_j) = \nu_{ij}$ interpreted as the certainty degree up to which x_i is non-preferred to x_j.

Notice that in [21] it has been proved that there exists a one-to-one correspondence between the set of reciprocal intuitionistic fuzzy preference relations and the set of asymmetric fuzzy preference relations, and so the Consistency measures above can be directly applied to the case of IRFPRs.

1) Expert's degree of confidence: Given a reciprocal intuitionistic fuzzy preference relation, the hesitancy degrees used to define confidence measures at its three different levels: pair of alternatives, alternatives and relation levels, can be defined as follows:

Definition 8. *Given a reciprocal intuitionistic fuzzy preference relation $B = (b_{ij}) = (\langle \mu_{ij}, \nu_{ij} \rangle)$, the* confidence level associated to the intuitionistic preference value b_{ij} *is measured as*

$$CFL_{ij} = 1 - \tau_{ij},$$

with τ_{ij} being the hesitancy degree associated to b_{ij}.

As noted before in Section II-B, $\tau_{ij} = 1 - \mu_{ij} - \nu_{ij}$ and therefore we have that $CFL_{ij} = \mu_{ij} + \nu_{ij}$. In other words, when $CFL_{ij} = 1$ ($\mu_{ij} + \nu_{ij} = 1$) then $\tau_{ij} = 0$ and there is no hesitation at all. The lower the value of CFL_{ij}, the higher the value of τ_{ij} and the more hesitation is present in the intuitionistic value b_{ij}.

Definition 9. *Given a reciprocal intuitionistic fuzzy preference relation $B = (b_{ij}) = (\langle \mu_{ij}, \nu_{ij} \rangle)$, the* confidence level associated to the alternative x_i *is defined as*

$$CFL_i = \frac{\sum_{\substack{j=1 \\ i \neq j}}^{n} \Big(CFL_{ij} + CFL_{ji} \Big)}{2(n-1)}.$$

Because B is reciprocal, we have that $CFL_{ij} = CFL_{ji}$ $(\forall i, j)$ and therefore it is

$$CFL_i = \frac{\sum_{\substack{j=1 \\ i \neq j}}^{n} CFL_{ij}}{n-1}.$$

A similar interpretation of CFL_i with respect to the confidence on the preference values on the alternative x_i can be done as it was done above with CFL_{ij}.

Definition 10. *The* confidence level associated to a reciprocal intuitionistic fuzzy preference relation $B = (b_{ij}) = (\langle \mu_{ij}, \nu_{ij} \rangle)$ *is measured as*

$$CFL_B = \frac{\sum_{i=1}^{n} CFL_i}{n}.$$

Notice that when $CFL_B = 1$, then the reciprocal intuitionistic fuzzy preference relation B is a reciprocal fuzzy preference relation.

III. Confidence-Consistency based Consensus Model IfRPRs

In many decision making processes it could be expected to associate a higher importance degree to the experts that provides both the more coherent or consistent answer and also the ones that present the highest degree of confidence with the provided solutions. In this section we present a new consensus approach that takes both experts degree of confidence and cosistency to aggregate the experts opinion. To do so the CC-IOWA operator proposed in [21] to fuse the experts opinions is used. This operator trades off consistency and confidence criteria in both re-ordering the preferences to aggregate and deriving the aggregation weights to use in their fusing to derive the collective preference. Once the collective IFPR is obtained, a proximity index (PI) measuring the level of agreement between the individual and collective preferences is computed. The consensus degree is defined taking into account both the Confidence and consistency leves and PI. When the consensus level reaches a threshold value, agreed by the group of experts, the resolution process of the GDM is carried out; otherwise a feedback mechanism is activated, and some personalised recommendations are generated to support the individual experts, until the threshold level of consensus is achieved. The feedback recommendations will help the experts to identify the preference values that should be considered for changing. The recommendations will also include the values the experts should use to increase the level of agreement in a consistent way.

A. Computing Proximity Indexes

The proximity degrees will measure how close the individual preferences are from the group or collective preferences. The collective preferences are obtained by fusing all the individuals' preferences using the confidence-consistency induced ordered weighted averaging (CC-IOWA) operator:

Definition 11 (CC-IOWA operator)**.** *Let a set of experts, $E = \{e_1, \ldots, e_m\}$, provide preferences about a set of alternatives, $X = \{x_1, \ldots, x_n\}$, using the reciprocal intuitionistic fuzzy preference relations, $\{B^1, \ldots, B^m\}$. A consistency and confidence IOWA (CC-IOWA) operator of dimension m, Φ_W^{CC}, is an IOWA operator whose set of order inducing values is the set of consistency/confidence index values, $\{CCI^1, \ldots, CCI^m\}$, associated with the set of experts.*

Therefore, the collective reciprocal intuitionistic fuzzy preference relation $B^{cc} = (b_{ij}^{cc}) = (\langle \mu_{ij}^{cc}, \nu_{ij}^{cc} \rangle)$ is computed as follows:

$$\mu_{ij}^{cc} = \Phi_W^{CC}\left(\langle CCI^1, \mu_{ij}^1 \rangle, \cdots, \langle CCI^m, \mu_{ij}^m \rangle\right) = \sum_{h=1}^{m} w_h \cdot \mu_{ij}^{\sigma(h)} \tag{3}$$

$$\nu_{ij}^{cc} = \Phi_W^{CC}\left(\langle CCI^1, \nu_{ij}^1 \rangle, \cdots, \langle CCI^m, \nu_{ij}^m \rangle\right) = \sum_{h=1}^{m} w_h \cdot \nu_{ij}^{\sigma(h)} \tag{4}$$

$$CCI^h = (1-\delta) \cdot CL^h + \delta \cdot CFL^h \tag{5}$$

such that $CCI^{\sigma(h-1)} \geq CCI^{\sigma(h)}$, $w_{\sigma(h-1)} \geq w_{\sigma(h)} \geq 0$ ($\forall h \in \{2, \cdots, m\}$) with $\sum_{h=1}^{m} w_h = 1$, CL_{ij}^h the consistency level associated to $R^h = F(B^h)$, CFL^h the confidence level associated to B^h, and $\delta \in [0,1]$ a parameter to control the weight of both consistency and confidence criteria in the inducing variable.

The general procedure for the inclusion of importance weight values, $\{u_1, \ldots, u_m\}$, in the aggregation process involves the transformation of the values to aggregate under the importance degree to generate a new value and then aggregate these new values using an aggregation operator. In the area of quantifier guided aggregations, Yager provided a procedure to evaluate the overall satisfaction of m important criteria (experts) by an alternative x by computing the weighting vector associated to an OWA operator as follows [30]:

$$w_h = Q\left(\frac{S(h)}{S(m)}\right) - Q\left(\frac{S(h-1)}{S(m)}\right)$$

being Q the membership function of the linguistic quantifier, $S(h) = \sum_{k=1}^{h} u_{\sigma(k)}$, and σ the permutation used to produce the ordering of the values to be aggregated. This approach for the inclusion of importance degrees associates a zero weight to those experts with zero importance degree. The linguistic quantifier is a Basic Unit-interval Monotone (BUM) function $Q : [0,1] \to [0,1]$ such that $Q(0) = 0$, $Q(1) = 1$ and if $x > y$ then $Q(x) \geq Q(y)$.

Yager extended this procedure to the case of IOWA operator. In this case, each component in the aggregation consists of a triple, with first element being the argument value to aggregate, the second element the importance weight value associated to the first element and the third element being the order inducing value [29]. The same expression as above is used with σ being the permutation that order the induce values from largest to lowest. In our case, we propose to use the consistency/confidence values associated with each expert both as an importance weight and as the order inducing values. Thus, the ordering of the preference values is first induced by the ordering of the experts from the most to the least consistent/confident, and the weights of the CC-IOWA operator is obtained as follows:

$$w_h = Q\left(\frac{\sum_{k=1}^{h} CCI^{\sigma(k)}}{T}\right) - Q\left(\frac{\sum_{k=1}^{h-1} CCI^{\sigma(k)}}{T}\right)$$

with $T = \sum_{k=1}^{m} CCI^k$.

The metric used to compute consistency indexes is used here to compute the proximity (similarity) between an individual IFPR, $R^h = (r_{ij}^h)$, and the collective IFPR, $R^c = (r_{ij}^c)$, at the three different levels of the relation:

Level 1. *Proximity index on pairs of alternatives*. The proximity of an expert, e_h, preference value on the pair of alternatives (x_i, x_k) to the group one, denoted PP_{ik}^h, is defined as:

$$PP_{ij}^h = 1 - d(r_{ij}^h, r_{ij}^c)$$

Level 2. *Proximity index on alternatives*. The proximity of an expert, e_h, preferences involving the alternative x_i to the group ones, denoted PA_i^h, is defined as:

$$PA_i^h = \frac{\sum_{j=1; j\neq i}^{n} (PP_{ij}^h + PP_{ji}^h)}{2(n-1)}$$

Level 3. *Proximity index on the relation*. The proximity of an expert, e_h, preference relation to the group one, denoted PI^h, is defined as:

$$PI^h = \frac{\sum_{i=1}^{n} PA_i^h}{n}$$

B. Computing Consensus Levels

Given an IFPR, R, its consensus level (CL) is defined as follows:

$$CL = \delta \cdot CCL + (1-\delta) \cdot PI \tag{6}$$

where $\delta \in [0,1]$ is a parameter to control the weight of both, on the one hand the Consitency-Confidence Criteria and on the other hand the proximity criteria. Similar expressions apply to CL_i and CL_{ij}, respectively. A value of $\delta > 0.5$ is used to provide more importance to the consistency-confindence index in the computation of the consensus degrees. The particular value to use will obviously depend on the group of experts and the importance they would like to allocate to the consistency and the confidence of each expert, but we can assume that the threshold value $\gamma \in [0.5, 1)$.

The consensus levels can be used to decide whether the feedback mechanism should be applied or not to give advice to the experts, or when the consensus reaching process has to come to an end. When $CL^h \, (h = 1, \ldots, m)$ satisfies a minimum satisfaction threshold value $\gamma \in [0.5, 1)$, then the consensus reaching process ends, and the selection process is applied to achieve the solution of consensus.

C. Feedback Mechanism

When at least one of the experts' consensus levels is below the fixed threshold value, a feedback mechanism is activated to generate personalised advice to those experts. This activity includes two steps: *Identification of the preference values* that should be changed and *Generation of advice*.

1) Identification of the Preference Values: The preference values that are contributing less to the consensus are identified. To do that, the following three step identification procedure that uses the proximity and consistency indexes is carried out:

Step 1. The experts with a consensus level lower than the threshold value γ are identified:

$$EXPCH = \{h \mid CL^h < \gamma\}.$$

Step 2. For the identified experts, their alternatives with a consensus level lower than the satisfaction threshold γ are identified:

$$ALT = \{(h,i) \mid e_h \in EXPCH \;\&\; CL_i^h < \gamma\}.$$

Step 3. Finally, the preference values to be changed are:

$$APS = \{(h,i,k) \mid (h,i) \in ALT \;\&\; CL_{ik}^h < \gamma\}.$$

2) Generation of Advice: The feedback mechanism generates personalised recommendation rules, which will tell the experts the preference values they should change and the new preference values to use in order to increase their consensus level. For all $(h,i,j) \in APS$, the personalised recommendation rules are identified as follow:

1) If $(i,j) \in EV^h$ the recommendation generated for expert e_h is: *"You should change your preference value for the pair of alternatives (i,j), $r_{ij}^h = \left\langle \mu_{ij}^h, \nu_{ij}^h \right\rangle$, to a value closer to $rr_{ij}^h = \left\langle r\mu_{ij}^h, r\nu_{ij}^h \right\rangle$."*
2) If $(i,j) \in MV^h$ the recommendation generated for expert e_h is:*"Your missing preference value for the pair of alternatives (i,j) should be as close as possible to $rr_{ij}^h = \left\langle r\mu_{ij}^h, r\nu_{ij}^h \right\rangle$."*

$$\left\langle r\mu_{ij}^h, r\nu_{ij}^h \right\rangle = \left\langle \delta \cdot \mu_{ij}^h + (1-\delta)\cdot \mu_{ij}^c, \quad \delta \cdot \nu_{ij}^h + (1-\delta)\cdot \nu_{ij}^c \right\rangle$$

D. Ranking of alternatives by means of the Fuzzy Non-dominance Degree for Intuitionistic Fuzzy Preference Relations

Let $B = (b_{ij})$ with $b_{ij} = \langle \mu_{ij}, \nu_{ij} \rangle$ be an IFPR. It has been proved that two FPRs can be associated to the IFPR:

- The asymmetric FPR: $R = (r_{ij}) = (\mu_{ij})$.
- The score FPR: $P = (p_{ij}) = (S_{WC}(b_{ij}))$.

Notice that in preference modelling, given an asymmetric FPR, it is always possible to derive a reciprocal FPR. When this procedure is applied, P is the reciprocal FPR that derives from R.

A procedure to rank alternatives assessed via an IFPR B could therefore be performed by applying the fuzzy quantifier guided non-dominance degree associated to its FPRs . In [6] it has been proved that the fuzzy quantifier guided non-dominance degree obtained via FPR can be extended to te case of IFPRs via the isomorphism proved in [21].

Definition 12. *Let $B = (b_{ij})$ with $b_{ij} = \langle \mu_{ij}, \nu_{ij} \rangle$ be an IFPR. The fuzzy quantifier guided non-dominance degree of alternative x_i measures the degree up to which such alternative is not dominated by a fuzzy majority of the remaining alternatives, and it is expressed as follows:*

$$IQGNDD_i = \phi_Q\left(1 - \mu^s_{ji}, j = 1, \ldots, n\right),$$

with $\mu^s_{ji} = \max\left\{\mu_{ji} - \mu_{ij}, 0\right\}$.

IV. Conclusions

Nowadays the complexity of decision making situations makes that experts present high degrees of uncertainty when expressing their opinions and judgments. To overcome these situations new GDM methodologies that are able to cope with this uncertainty and also asses the experts degree of confidence with the answer provided are becoming more than necessary. In this contribution we present a new consensus model that takes advantage of the IFPRs to allow the experts to allocate their uncertainty while expressing their opinions and to asses their degree of confidence inherent with their opinions. The proposed approach presents a procedure based on the experts degree of consistency and confidence to fuse their opinions and provide advice or recommendations for the experts to bring their opinions closer. Moreover in order to provide a ranking of the alternatives a new Non Dominance Intuitionistc Fuzzy operator has been included.

V. Acknowledgments

The authors would like to acknowledge the support from FEDER funds in the FUZZYLING-II Project TIN2010-17876, as well as the support from the Andalusian Excellence Projects TIC-05299 and TIC-5991.

References

[1] K. T. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1):87–96, 1986.

[2] J.C. Bezdek, B. Spillman, and R. Spillman R. A fuzzy relation space for group decision theory. *Fuzzy Sets and Systems*, 1(4):255–268, 1978.

[3] F. J. Cabrerizo, R. Ureña, W. Pedrycz, and E. Herrera-Viedma. Building consensus in group decision making with an allocation of information granularity. *Fuzzy Sets and Systems*, 255:115–127, 2014.

[4] F. Chiclana, E. Herrera-Viedma, S. Alonso, and F. Herrera. Cardinal consistency of reciprocal preference relations: A characterization of multiplicative transitivity. *IEEE Transactions on Fuzzy Systems*, 17(1):14–23, 2009.

[5] F. Chiclana, F. Mata, E. Herrera-Viedma, and L. MartÃnez. Integration of a consistency control module within a consensus decision making model. *International Journal of Uncertainty, Fuzziness and Knowledge Based Systems*, 16(1):35 – 53, 2008.

[6] F. Chiclana, R. Urena, and E. Herrera-Viedma. Choice degrees in decision-making: A comparison between intuitionistic and fuzzy preference relations approaches. In *Fuzzy IEEE 2016*.

[7] P.C. Fishburn. *Utility theory for decision making*. Krieger, Melbourne, FL, 1979.

[8] Hamido Fujita, Masaki Kurematsu, and Jun Hakura. Virtual doctor system (vds) and ontology based reasoning for medical diagnosis. In Endre Pap, editor, *Intelligent Systems: Models and Applications*, volume 3 of *Topics in Intelligent Engineering and Informatics*, pages 197–214. Springer Berlin Heidelberg, 2013.

[9] Debashree Guha and Debjani Chakraborty. A new approach to fuzzy distance measure and similarity measure between two generalized fuzzy numbers. *Applied Soft Computing*, 10(1):90 – 99, 2010.

[10] E. Herrera-Viedma, S. Alonso, F. Chiclana, and F.Herrera. A consensus model for group decision making with incomplete fuzzy preference relations. *IEEE Transactions on Fuzzy Systems*, 15(5):863–877, 2007.

[11] E. Herrera-Viedma, F. Chiclana, F.Herrera, and S. Alonso. Group decision-making model with incomplete fuzzy preference relations based on additive consistency. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 37(1):176–189, 2007.

[12] E. Herrera-Viedma, F. Herrera, F. Chiclana, and M. Luque. Some issues on consistency of fuzzy preference relations. *European Journal of Operational Research*, 154(1):98–109, 2004.

[13] V.B Hinsz. Cognitive and consensus processes in group recognition memory performance. *Journal of Personality and Social Psychology*, 59:705–718, 1990.

[14] H. H. Johnson and J. M. Torcivia. Group and individual performance on a single stage task as a function of distribution of individual performance. *Expert Systems with Applications*, (3):266–263, 1967.

[15] I. Millet. The effectiveness of alternative preference elicitation methods in the analytic hierarchy process. *Journal of Multi-Criteria Decision Analysis*, 6(1):41–51, 1997.

[16] M. Roubens and P. Vincke. *Preference modeling*. Springer, Berlin, 1985.

[17] T. L. Saaty. *The analytic hierarchy process*. McGraw-Hill, 1980.

[18] Geoffrey M. Stephenson, Dominic Abrams, Wolfgang Wagner, and Gillian Wade. Partners in recall: Collaborative order in the recall of a police interrogation. *British Journal of Social Psychology*, 25(4):341–343, 1986.

[19] E.E. Szmidt and J. Kacprzyk. Using intuitionistic fuzzy sets in group decision making. *Control and Cybernetics*, 31(4):1037–1053, 2002.

[20] T. Tanino. Fuzzy preference orderings in group decision making. *Fuzzy sets and system*, 12:117–131, 1984.

[21] Raquel Urena, Francisco Chiclana, Hamido Fujita, and Enrique Herrera-Viedma. Confidence-consistency driven group decision making approach with incomplete reciprocal intuitionistic preference relations. *Knowledge-Based Systems*, 89:86 – 96, 2015.

[22] Ioannis K. Vlachos and George D. Sergiadis. Intuitionistic fuzzy information - applications to pattern recognition. *Pattern Recognition Letters*, 28(2):197 – 206, 2007.

[23] J. Wu and F. Chiclana. Non-dominance and attitudinal prioritisation methods for intuitionistic and interval-valued intuitionistic fuzzy preference relations. *Expert Systems with Applications*, 39(18):13409–13416, 2012.

[24] Jian Wu and Francisco Chiclana. Multiplicative consistency of intuitionistic reciprocal preference relations and its application to missing values estimation and consensus building. *Knowledge-Based Systems*, 71(0):187 – 200, 2014.

[25] Z. Xu. Information science. *International Journal of Intelligent Systems*, 177(9):2363–2379, 2007.

[26] Z. Xu, X. Cai, and E. Szmidt. Algorithms for estimating missing elements of incomplete intuitionistic preference relations. *International Journal of Intelligent Systems*, 26(9):787–813, 2011.

[27] Z. S. Xu. A survey of preference relations. *International Journal of General System*, 27(36):179–203, 2007.

[28] Zeshui Xu, Jian Chen, and Junjie Wu. Clustering algorithm for intuitionistic fuzzy sets. *Information Sciences*, 178(19):3775 – 3790, 2008.

[29] R. R. Yager. Induced aggregation operators. *Fuzzy Sets and Systems*, 137:59–69, 2003.

[30] Ronald R. Yager and Alexander Rybalov. Uninorm aggregation operators. *Fuzzy Sets and Systems*, 80(1):111 – 120, 1996. Fuzzy Modeling.

[31] Paul Zarnoth and Janet A. Sniezek. The social influence of confidence in group decision making. *Journal of Experimental Social Psychology*, 33(4):345 – 366, 1997.